high-pass filter, we observe that the shape of the output spectra is slightly affected by the glider dynamics. However, in Fig. 3 the load-factor output presents a peak at the phugoid frequency, which is significant in spite of the fact that the turbulent energy is rather low in the range of frequencies of the characteristic modes of the aircraft. In this respect, the extension of the analysis to include a low-altitude turbulence model would probably lead to higher output energies for n. The same figure reports the limit of validity of the zero and first-order approximations and shows that the use of a zero-order model is satisfactory for wave numbers lower than 10^{-2}, but causes a 35% underestimate of the output of the load-factor power at resonance.

References

[1]de Matteis, G., "Dynamics of Hang-Gliders," *Journal of Guidance, Control, and Dynamics,* Vol. 14, No. 6, 1991, pp. 1145–1152.

[2]de Matteis, G., "Response of Hang-Gliders to Control," *Aeronautical Journal,* Vol. 94, No. 938, 1990, pp. 289–294.

[3]Jones, R. T., "Dynamics of Ultralight Aircraft: Motion in Vertical Gusts," NASA TM X-73228, April 1977.

[4]Kroo, I. M., "Aerodynamics, Aeroelasticity and Stability of Hang-Gliders," Ph.D. Thesis, Stanford Univ., Stanford, CA, 1983.

[5]Etkin, B., *Dynamics of Atmospheric Flight,* Wiley, New York, 1972, Chaps. 5, 13.

[6]Etkin, B., "Turbulent Wind and Its Effect on Flight," *Journal of Aircraft,* Vol. 18, No. 5, 1981, pp. 327–345.

[7]Etkin, B., "Theory of the Flight of Airplanes in Isotropic Turbulence—Review and Extension," AGARD Rept. 372, April 1961.

Thrust/Speed Effects on Long-Term Dynamics of Aerospace Planes

Gottfried Sachs*
Technische Universität München, 8000 Munich, Germany

Nomenclature

C_D = drag coefficient
C_L = lift coefficient
C_m = pitching moment coefficient
C_{m_i} = $2\partial C_m/\partial(i\bar{c}/V_0)$ where $i = q, \dot{\alpha}$
C_{N_V} = $\partial C_N/\partial(V/V_0)$ where $N = D, L, m$
C_{N_α} = $\partial C_N/\partial\alpha$ where $N = D, L, m$
C_{N_δ} = $\partial C_N/\partial\delta$ where $N = D, L, m$
$\bar{c}$ = mean aerodynamic chord
g = acceleration due to gravity
h = altitude
i_y = radius of gyration
k_ρ = $-g/(V_0^2\rho_h)$
M = Mach number
m = mass
n_h = thrust/altitude dependence, $n_h = (1/\rho_h)\partial(T/T_0)/\partial h$
n_V = thrust/speed dependence, $n_V = (V_0/T_0)\partial T/\partial V$
R = radius of the Earth
S = reference area
s_h = height mode root
T = thrust
V = speed
z_T = thrust-line offset
α = angle of attack (α_T for thrust)
Δ = denoting a perturbation, e.g., ΔV
δ = control (δ_e for pitch, δ_T for thrust)
μ = relative mass parameter, $\mu = 2m/(\rho S\bar{c})$
ρ = air density
ρ_h = density gradient, $\rho_h = (1/\rho)\mathrm{d}\rho/\mathrm{d}h$
σ = real part of a complex variable (σ_N for a zero, σ_p for phugoid)
ω_n = absolute value of a complex variable (ω_{n_N} for a zero, ω_{n_p} for a phugoid)

Received April 30, 1991; revision received June 10, 1991; accepted for publication June 24, 1991.

*Professor of Flight Mechanics and Flight Control. Associate Fellow AIAA.

Introduction

In recent papers,[1-6] the effect of thrust changes due to speed on long-term dynamics of vehicles in supersonic and hypersonic flight has been considered. The renewed interest in supersonic and hypersonic flight dynamics problems is the result of current aerospace plane programs in various countries like the German Sänger and the U.S. National Aerospace Plane (NASP) programs as well as others. It may also be stimulated by a better knowledge of the characteristics of propulsion systems proposed for hypersonic vehicles. Advanced air-breathing propulsion systems considered for application in supersonic and hypersonic flight are rather complex as regards their dependence of thrust on speed or Mach number.[1] They show significant variations in thrust throughout the Mach number range. These variations are due to the characteristics of an individual engine type used in a certain Mach number range like a turbojet, a ramjet, or a scramjet. They are also associated with changes in engine cycles when converting from one engine type to another.

For long-term dynamics, two types of thrust/speed effects are of importance. One may be termed a direct effect, which means thrust influence on force characteristics alone due to the change of thrust with speed. The other, which may be termed an indirect effect, is concerned with the influence of thrust on pitching moment due to thrust-axis offset and interaction with the flowfield. The direct effect of thrust/speed dependence has been considered in detail. For supersonic flight, it has been shown that this effect on the phugoid is small or even negligible.[5,6] By contrast, the height mode is rather sensitive as regards direct thrust/speed effects.[1-6] For indirect thrust/speed effects, pitching moment/speed sensitivity due to thrust-axis offset is considered to have an important long-period influence that may be significant even when the direct thrust/speed effects are small.[7,8]

The purpose of this Note is to provide more insight into thrust/speed effects on long-term modes of motion and to present new results for the hypersonic flight regime. Analytical considerations are presented to obtain results of a general nature. For both direct and indirect thrust/speed effects, it will be shown that there are significant differences in the sensitivities of the phugoid and height mode as regards such effects.

Aerospace Plane Dynamics

A thrust change with speed results in a force deviation from trim. In linearized form, thrust change may be expressed as

$$\Delta T/T_0 = n_V \Delta V/V_0 \tag{1}$$

In case of a thrust-axis offset z_T, an aerodynamic pitching moment $(M_0)_{\text{aero}} = -T_0 z_T$ is necessary for trim. As a consequence, a deviation in pitching moment results when the trim state is disturbed. This indirect thrust/speed effect may be formulated as a nondimensional stability derivative

$$C_{m_V} = (n_V - 2)(z_T/\bar{c})C_D/\cos\alpha_T \tag{2}$$

There may be additional thrust effects like an influence on the flowfield. They are considered to be included in C_{m_V}.

With the use of the expressions in Eqs. (1) and (2) for describing direct and indirect thrust/speed effects, the linearized equations of the longitudinal motion for a horizontal reference flight condition at hypersonic speed may be expressed as

$$A(s)x(s) = B\delta(s) \tag{3a}$$

where s is the Laplace operator and (with the reference time $\tau = \bar{c}/V_0$)

$$A(s) = \begin{bmatrix} \mu s\tau + (2 - n_V)C_D + C_{D_V} & C_{D_\alpha} + C_D \tan\alpha_T & \mu s\tau^2 g/V_0 + \bar{c}\rho_h(C_D(1 - n_h) \\ 2C_L + C_{L_V} + 2\mu\bar{c}/R & C_{L_\alpha} + C_D & -\mu(s\tau)^2 + \bar{c}\rho_h(C_L + n_h C_D \tan\alpha_T) \\ + n_V C_D \tan\alpha_T & & + \mu(\tau^2/R)(2g - V_0^2/R) \\ \mu s\tau(i_y/\bar{c})^2\bar{c}/R & -\mu(s\tau)^2(i_y/\bar{c})^2 & -\mu(s\tau)^3(i_y/\bar{c})^2 + (s\tau)^2 C_{m_q}/2 \\ + C_{m_V} & + s\tau(C_{m_q} + C_{m_{\dot{\alpha}}})/2 + C_{m_\alpha} & -\mu s\tau(i_y/R)^2 \end{bmatrix} \tag{3b}$$

$$B = -\begin{bmatrix} C_{D_\delta} & -C_D \\ C_{L_\delta} & C_D \tan\alpha_T \\ C_{m_\delta} & (z_T/\bar{c})C_D/\cos\alpha_T \end{bmatrix} \tag{3c}$$

$$x(s) = [\Delta V/V_0 \quad \Delta\alpha \quad \Delta h/\bar{c}]^T \qquad \delta(s) = [\delta_e \delta_T]^T \tag{3d}$$

There are five roots of the characteristic equation that is a quintic in hypersonic flight due to the effect of altitude dependent forces rather than a quartic, i.e., $s^5 + Bs^4 + Cs^3 + Ds^2 + Es + F = 0$. Three modes of motion are associated with the roots addressed. For long-term dynamics, the phugoid and height mode are of interest. They will be considered in the following.

Direct Thrust/Speed Effects

Direct thrust/speed effects, that are understood here as an influence of thrust on force characteristics alone, are described first. For this case, phugoid damping and height mode stability characteristics are considered to be influenced primarily whereas phugoid frequency is rather insensitive. In Refs. 5 and 6 it was pointed out for the supersonic flight regime that phugoid damping is dependent on direct thrust/speed changes to a small extent only. This effect is reduced further when increasing speed so that it can be ignored in the hypersonic flight regime. An example is presented in Fig. 1, which illustrates the insensitivity of phugoid damping (σ_p) as regards thrust/speed dependence (described by n_V). By contrast, phugoid damping is rather sensitive to thrust changes with altitude (described by n_h in Fig. 1).

The second direct thrust/speed effect concerns the height mode that is generally considered to be strongly influenced. This holds for the lower range of hypersonic Mach numbers (Fig. 1). However, thrust/speed effect on the height mode is significantly reduced in the high hypersonic flight regime as also shown in Fig. 1. This reduction is associated with the curvature of the Earth and the related centrifugal force that gains an influence no longer negligible on long-term dynamics at very high speed. As regards thrust dependence on altitude n_h, the height mode shows a significant sensitivity similar to phugoid damping. By contrast to the thrust/speed effect, the thrust/altitude effect appears not to be reduced in the high hypersonic flight regime (Fig. 1).

The results presented in Fig. 1. can be confirmed by analytical expressions for approximations of phugoid damping σ_p and height mode eigenvalue s_h. By approximate factoring the characteristic equation, the following expressions may be derived:

$$\sigma_p \approx -(1 - n_h + k_p n_V)\frac{g}{V_0}\frac{C_D}{C_L}$$

$$s_h \approx 2\left[1 - n_h + \left(1 - \frac{V_0^2}{gR}\right)\left(\frac{n_V}{2} - 1\right)\right]\frac{g}{V_0}\frac{C_D}{C_L} \tag{4}$$

Equation (4) shows that the reduction of the thrust/speed effect n_V on the height mode s_h is caused by the term $V_0^2/(gR)$

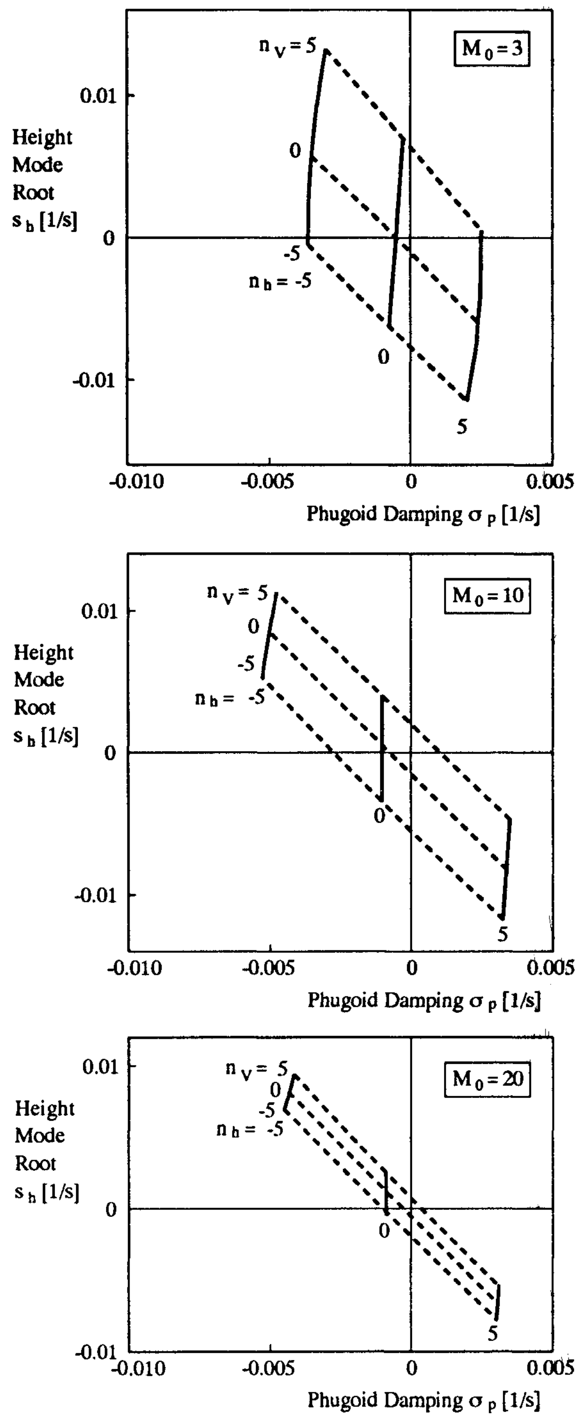

Fig. 1 Direct effects of thrust/speed dependence (n_V) and thrust/altitude dependence (n_h).

Table 1 Factor k_ρ as a function of Mach number (atmospheric data from Ref. 9), altitude range: $0 \le h \le 86$ km

Mach number	5	10	15	20
Maximal k_ρ	0.035	0.009	0.004	0.0021
Minimal k_ρ	0.026	0.007	0.003	0.0017

that represents a nondimensional form of the centrifugal force. This term gains an influence no longer negligible for Mach numbers greater than about $M_0 = 8$–10, and it approaches one for orbital speed. As a consequence, the parenthesized expression multiplying n_V is reduced with increasing Mach number and finally becomes zero.

Equation (4) also makes evident why thrust/speed effect on phugoid damping (σ_p) is practically nonexistent. This is because the thrust slope term n_V is multiplied by the factor

$$k_\rho = -g/(V_0^2 \rho_h) \tag{5}$$

This factor represents a small quantity valid throughout the whole Mach number and altitude range of interest. This is illustrated in Table 1, which also shows that k_ρ monotonically decreases with Mach number. As a consequence of the relation $k_\rho \ll 1$, the effect of n_V on phugoid damping is substantially reduced for the hypersonic flight regime where this reduction is larger the higher the Mach number is ($k_\rho \sim 1/V_0^2$).

Indirect Thrust/Speed Effects

Indirect thrust/speed effects are understood here as pitching moments produced by thrust changes with speed. They are generally considered to have a large influence on the long-term modes of motion. This influence may be larger than direct thrust/speed effects alone. Indirect thrust/speed effects are considered to produce large changes of phugoid frequency and damping (including unstable characteristics) as well as large changes of the height mode. It will be shown in the following that there are indirect thrust/speed effects that have characteristics specific for hypersonic flight and that may differ from the influence usually considered to exist.

Some insight can be provided by analytical expressions for approximations of the eigenvalues. From factoring the characteristic equation including the stability derivative C_{m_V}, the following approximations valid for hypersonic flight may be derived:

$$\omega_{n_p} \approx \omega_{n_p}^* \sqrt{1 - k_\rho \frac{C_{m_V}}{C_L \partial C_m/\partial C_L}}, \qquad \sigma_p \approx \sigma_p^*$$

$$s_h \approx s_h^* + \left(1 - \frac{V_0^2}{gR}\right) \frac{C_{m_V}}{C_L \partial C_m/\partial C_L} \frac{\partial C_D}{\partial C_L} \frac{g}{V_0} \tag{6}$$

with $\omega_{n_p}^*$, σ_p^*, and s_h^* denoting the reference case when $C_{m_V} = 0$; see also Eq. (4).

These approximations may be used to describe the tendency of the C_{m_V} effect and to provide an indication of the sensitivities of the phugoid and the height mode. For phugoid frequency as described in Eq. (6), it may be seen that C_{m_V} is multiplied by the small factor $k_\rho \ll 1$. This means that the indirect thrust/speed effect C_{m_V} on the phugoid is also reduced in hypersonic flight.

It may be of interest to note that both direct and indirect thrust/speed effects on the phugoid are reduced in the same way. Because of $k_\rho \ll 1$, this reduction holds throughout the whole hypersonic flight regime.

According to Eq. (6), the height mode is influenced rather strongly by indirect thrust speed effects. However, a reduction of these effects also exists for high hypersonic Mach numbers. This is again due to the centrifugal force term $V_0^2/(gR)$ that reduces the parenthesized expression by which C_v is multiplied, Eq. (6).

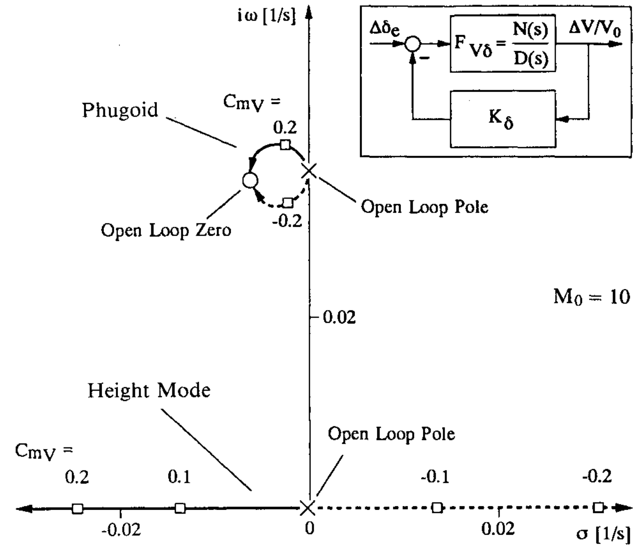

Fig. 2 Indirect thrust/speed effects (C_{m_V}) on phugoid and height mode, $M_0 = 10$.

The results presented for the height mode show that there is also a reduction both for direct as well as indirect thrust/speed effects. This is valid for high hypersonic Mach numbers. At low hypersonic Mach numbers, the height mode is rather sensitive to such effects.

The tendencies as described by the previous approximations can be confirmed and more insight may be provided by analytical considerations concerning the root locus technique where C_{m_V} is treated as the gain in an appropriate hypothetical feedback loop. This is illustrated in the right side of Fig. 2 where the transfer function $F_{V\delta}$ between speed V and pitch control deflection δ_e and a control loop with feedback of V to δ_e is shown. The gain of the control loop K_δ can be related to C_{m_V} according to

$$C_{m_V} = -K_\delta C_{m_\delta} \tag{7}$$

Considering C_{L_δ} and C_{D_δ} as zero, a control loop with speed feedback to pitch control exerts the same effect on aerospace plane dynamics as C_{m_V} according to Eq. (7). This means that the root locus for this control loop can also be used for describing the effect of C_{m_V}.

The root locus as influenced by the gain K_δ is determined by the relative location of the poles and zeros of the open-loop transfer function. For long-term dynamics, the phugoid and height mode considered earlier represent the open-loop poles. There are two zeros $\sigma_N \pm i\sqrt{\omega_{n_N}^2 - \sigma_N^2}$ of the numerator $N(s)$. For hypersonic flight, the following approximations hold:

$$\sigma_N \approx (\sigma_p^*)_{n_h = 1} - \frac{1}{2} \frac{g/V_0}{\partial C_D/\partial C_L}, \qquad \omega_{n_N} \approx \omega_{n_p}^* \tag{8}$$

with σ_p^* and $\omega_{n_p}^*$ denoting the open-loop phugoid poles.

Equation (8) suggests a close relationship between the zeros and related phugoid poles. The close proximity of these two zeros to the open-loop phugoid poles reduces the effect of C_{m_V} on the closed-loop phugoid poles.

These characteristics of a general nature are illustrated in more detail in Fig. 2, which presents a numerical example. The main result concerns the phugoid. As may be seen, there is a close relationship between the open-loop zeros and related phugoid poles. As a consequence, the travel of the phugoid roots is blocked by nearby zeros. This means that the phugoid shows only little change with C_{m_V}. The blocking effect holds for positive as well as negative C_{m_V} values. Opposite to the little phugoid change, the height mode is strongly influenced (Fig. 2). The quantitative results presented suggest a (practi-

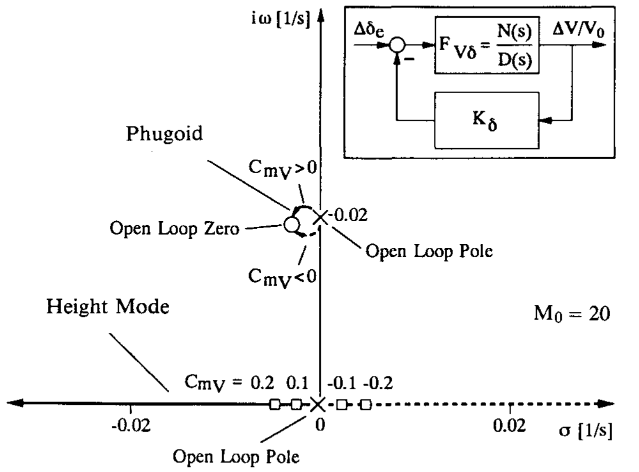

Fig. 3 Indirect thrust/speed effects (C_{m_V}) on phugoid and height mode, $M_0 = 20$.

cally) linear dependence of s_h on C_{m_V} for the range considered. This is in agreement with the approximation described in Eq. (6).

The effect of C_{m_V} is reduced for both long-term modes of motion at higher Mach numbers (Fig. 3). From this figure it follows that the indirect thrust/speed effects considered here show a reduction that is similar to the direct case (n_V) described in Fig. 1.

Conclusions

It is shown that direct thrust/speed effects have practically no influence on phugoid characteristics in hypersonic flight. The height mode is rather sensitive to this effect for lower hypersonic Mach numbers. At high hypersonic Mach numbers, the direct thrust/speed effect on the height mode is also reduced. By contrast, direct thrust/altitude effects are significant on both the phugoid and height mode. This holds throughout the hypersonic Mach number range.

Indirect thrust/speed effects are concerned with an influence on pitching moment due to thrust-axis offset. In hypersonic flight, this type of thrust/speed effect is significantly reduced as regards the phugoid. In particular, it may not cause aperiodic phugoid instability that is usually considered to be introduced when this effect is beyond a critical value. Rather, the phugoid exists as an oscillation, the stability of which may even be increased. However, the height mode is sensitive to indirect thrust/speed effects. For this mode of motion, aperiodic instability may be caused by negative values of pitching moment variation with speed due to thrust effects. For positive values, height mode stability is increased. At high hypersonic Mach numbers, indirect thrust/speed effects on the height mode are reduced.

In summary, direct and indirect thrust/speed effects are reduced in hypersonic flight. For the phugoid, this is valid throughout the whole hypersonic flight regime. There is also a reduced sensitivity of the height mode. This reduction in thrust/speed effects is valid for high hypersonic Mach numbers.

References

[1]Berry, D. T., "National Aerospace Plane Longitudinal Long-Period Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 1, 1991, pp. 205–206.

[2]Stengel, R. F., "Comments on 'Effect of Thrust/Speed Dependence on Long-Period Dynamics in Supersonic Flight'," *Journal of Guidance, Control, and Dynamics* (to be published).

[3]Markopoulos, N., Mease, K. D., and Vinh, N. X., "Thrust Law Effects on the Long-Period Modes of Aerospace Craft," AIAA Paper 89-3379, Boston, MA, Aug. 1989.

[4]Markopoulos, N., and Mease, K. D., "Thrust Law Effects on the Longitudinal Stability of Hypersonic Cruise," AIAA Paper 90-2820, Portland, OR, Aug. 1990.

[5]Sachs, G., "Effect of Thrust/Speed Dependence on Long-Period Dynamics in Supersonic Flight," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 1163–1166.

[6]Sachs, G., "Reply by Author to Robert F. Stengel," *Journal of Guidance, Control, and Dynamics* (to be published).

[7]Stengel, R. F., "Altitude Stability in Supersonic Cruising Flight," *Journal of Aircraft*, Vol. 7, No. 5, 1970, pp. 464–473.

[8]Sachs, G., "The Effect of Pitching-Moments on Phugoid and Height Mode in Supersonic Flight," *Journal of Aircraft*, Vol. 9, No. 3, 1972, pp. 252–254.

[9]Committee for Extension to the Standard Atmosphere, *U.S. Standard Atmosphere, 1976*, U.S. Government Printing Office, Washington, DC, 1976.

Frequency Analysis of the Hoop-Column Antenna Using a Simplified Model

Peter Bofah* and Ajit K. Choudhury†
Howard University, Washington, D.C. 20059

Introduction

BECAUSE of its light weight, the hoop-column antenna is a candidate for future space flights of large flexible structures. During the past decade, a great deal of research activity, both theoretical and experimental, was directed toward developing a lightweight space antenna for future large flexible structures. The hoop-column antenna and the wrap-rib antenna are the two antennas being investigated at the present time. The hoop-column antenna is described in Refs. 1 and 2 (see Fig. 1). In this Note, we develop a simple model of the hoop-column antenna and compare the frequencies generated by the simple model with that of a complex finite element model.[3] Our frequencies compare favorably with those predicted by the more complex finite element model.[3]

Physical Model of the Hoop-Column Antenna

The hoop-column antenna basically consists of a shallow parabolic reflector made of membrane-like material supported by concentric hoops. The center of the reflector is fixed by a column (feed mast) that also supports the reflector at various points by a system of upper and lower cables (see Fig. 1). In this Note, we present a simplified model of the hoop-column antenna. The frequencies obtained from the model are compared with those of the more complex NASTRAN finite element model.[3]

The simplified model (see Figs. 2) of the hoop-column antenna consists of a circular membrane reflector and is based on the following assumptions:

1) The central annular radius is fixed by the column.
2) The column is rigid.
3) The support cables at the top and the bottom of the antenna (reflector) are modeled as two sets of massless identical springs arranged so that they are 120 deg apart around the circumference of the membrane (see Figs. 2).

This model differs from the classical problem of the vibration of a circular membrane in the following respects:

1) The present model is annular.
2) In the present model, the outer boundary (hoop) of the annular membrane is not fixed but moves subject to the constraints of the springs and the inner boundary is fixed by the column.

Received Aug. 31, 1990; revision received Jan. 23, 1991; accepted for publication Feb. 18, 1991.

*Assistant Professor, Electrical Engineering Department.
†Associate Professor, Electrical Engineering Department.